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Economic control for generation in thermal power systems

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Abstract

This work is based on two proposed procedures for scheduling of generating units to obtain optimum economic dispatch with consideration of control over the generating units. Two mathematical models were built, taking the transmission losses into account. The steady state deviations in each model of frequency, mechanical power, electrical power and transmission losses in addition to outputs of generating units and the corresponding generation costs were analyzed, in each time interval, after the occurrence of the load change and damping of all synchronizing oscillations. The load change in each time interval has been taken, in the first model, as the difference between the value of the load in the present interval and its value in the previous one. In the second model it has been taken, optionally, as a certain specified constant value. Results were obtained for both models and have been compared with each other. © 1998 Elsevier Science Ltd. All rights reserved.

1. Introduction

Assume that the optimization time period (T) is divided into (n) intervals. The length of each time interval equals (Δt). In each time (t), the generation cost $(K_i)^t$ of the generating unit (i) can be expressed in a polynomial form as

$$(K_i)^t = a_i\left[(P_i)^t\right]^2 + b_i(P_i)^t + c_i \quad \$/\text{h} \tag{1}$$

where $(P_i)^t$ is the generated real power in time interval (t) and a_i, b_i and c_i are parameters of the generating unit (i).

The total generation cost (TK_T) of a power system containing (N) controllable generating units in the period (T) will be

$$(TK_T) = \sum_{t=1}^{n}\sum_{i=1}^{N}(K_i)^t(\Delta t) = \sum_{t=1}^{n}(K_T)^t(\Delta t) \quad \$ \tag{2}$$

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where $(K_T)^t$ is the total generation cost of the power system in interval (t), i.e.

$$(K_T)^t = \sum_{i=1}^{N} (K_i)^t.$$

The generated real power has the major influence on the cost function. The real generation of each generating unit can be raised by increasing the fuel input to increase the prime mover torque. A set of real generation powers must be selected that will minimize the cost function in each time interval.

With consideration of transmission losses, the power balance requires that the sum of the controlled generation powers must equal the load demand $(P_L)^t$ plus the transmission losses $(P_l)^t$ in each time interval (t). Therefore, we can obtain

$$\sum_{i=1}^{N} (P_i)^t = (P_L)^t + (P_l)^t. \tag{3}$$

Each generating unit must not be operated above its rating or below some minimum value according to the following inequalities

$$(P_{i\min}) \leqslant (P_i)^t \leqslant (P_{i\max}) \quad \text{for} \quad i = 1, 2, \ldots \text{ and } N \tag{4}$$

where $(P_{i\ \max})$ and $(P_{i\ \min})$ are the maximum and minimum power permitted for the generating unit (i), respectively.

Various techniques have been developed to solve the problem of power system economic dispatch and to allocate the system load on each generating unit [1–5]. The mathematical model, which has been built by the author in Ref. [6], is developed in this work to include the effect of the transmission losses on scheduling of the controlled generation powers for optimum economic dispatch. Therefore, two mathematical models were built to calculate the steady state deviations in each model of the system frequency, mechanical power and electrical power. The outputs of the generating units and their generation costs in addition to the transmission losses can be calculated, also, in each time interval. In the first model, the load change $(\Delta P_L)^t$ in each time interval (t) has been taken as the difference between the interval load and the load of the previous interval. In the second model, it has been suggested that the load change can be taken as an optional specified constant value $(\Delta P_{L\ \mathrm{sp}})$.

2. Mathematical model based on the time interval load change (model 1)

When the opposing torques (mechanical and electrical) acting on the rotating mass of a generating unit driven by a steam turbine are equal in magnitude, the rotational speed of the unit will be constant. If the electrical load is increased so that the electrical torque is larger than the mechanical torque, the rotating system will begin to slow. The mechanical torque must be increased to restore equilibrium. The speed is again held constant when the two torques become equal. The previous process must be repeated constantly due to the changes of the electrical load in a power system.

The difference between the deviation in mechanical power (ΔP_m) and electrical power (ΔP_e) related to the rate of change of a generating unit speed can be given by

$$(\Delta P_m) - (\Delta P_e) = M\,\mathrm{d}(\Delta\omega)/\mathrm{d}t \tag{5}$$

where (ω) is the rotational speed (rad/sec) and (M) is the angular momentum of the machine in Watts per radian per second per second.

The change in load $(\Delta P_L)'$ due to the change in system frequency (Δf) can be expressed by the following equation:

$$(\Delta P_L)' = D(\Delta f) \tag{6}$$

where (D) is defined as the change in load divided by the change in frequency.

The net change in the electrical power output is given by

$$(\Delta P_e) = (\Delta P_L) + (\Delta P_L)' = (\Delta P_L) + D(\Delta f). \tag{7}$$

The change in mechanical power input can be related to the change in frequency by the following equation:

$$(\Delta P_m) = -(1/R)(\Delta f) \tag{8}$$

where (R) is the ratio between the per unit change in frequency and the per unit change in output of the generating unit (i), and it equals the slope of the governor characteristic. Eqs. (5)–(8) are given in Ref. [7].

Now, to formulate the proposed mathematical model for a power system consisting of (N) generating units to supply a load $(P_L)^t$ in time interval (t), let there be a load change $(\Delta P_L)^t$. In steady state, after all synchronizing oscillations have damped, the frequency will be constant and equal to the same value for each unit, then

$$\left[\mathrm{d}(\Delta\omega)/\mathrm{d}t\right] = 0. \tag{9}$$

Substitute Eqs. (7)–(9) in Eq. (5) to obtain the following equation for unit (i) in interval (t):

$$-(1/R_i)(\Delta f)^t = (\Delta P_{Li})^t + D_i(\Delta f)^t, \quad i = 1, 2, \ldots \text{ and } N \tag{10}$$

where $(\Delta P_{Li})^t$ is the change in the controlled generation of unit (i). Taking the transmission losses into consideration, power balance requires that the sum of the changes in the controlled generation powers must equal the load change $(\Delta P_L)^t$ plus the change of transmission losses $(\Delta P_l)^t$, i.e.

$$\sum_{i=1}^{N}(\Delta P_{Li})^t = (\Delta P_L)^t + (\Delta P_l)^t. \tag{11}$$

Add Eq. (10) for the (N) generating units to obtain

$$-(\Delta f)^t\left[\sum_{i=1}^{N}(1/R_i) + \sum_{i=1}^{N} D_i\right] = \sum_{i=1}^{N}(\Delta P_{Li})^t. \tag{12}$$

The deviation in frequency $(\Delta f)^t$ corresponding to the load change $(\Delta P_L)^t$ can be obtained from Eqs. (11) and (12) as

$$(\Delta f)^t = -\left[(\Delta P_L)^t + (\Delta P_l)^t\right] \Big/ \left[\sum_{i=1}^{N}(1/R_i) + \sum_{i=1}^{N} D_i\right]. \tag{13}$$

Substitute Eq. (13) in Eq. (10) to obtain the change in the controlled generation $(\Delta P_{Li})^t$ of unit (i):

$$(\Delta P_{Li})^t = \left[(\Delta P_L)^t + (\Delta P_l)^t\right]\left[(1/R_i) + D_i\right] \Big/ \left[\sum_{i=1}^{N}(1/R_i) + \sum_{i=1}^{N} D_i\right]. \tag{14}$$

Assume that

$$S_i = (1/R_i) + D_i \tag{15}$$

and

$$S_T = \sum_{i=1}^{N}(1/R_i) + \sum_{i=1}^{N} D_i = \sum_{i=1}^{N} S_i. \tag{16}$$

Substitute Eq. (16) in Eq. (13) to obtain the change in frequency:

$$(\Delta f)^t = -\left[(\Delta P_L)^t + (\Delta P_l)^t\right]/S_T. \tag{17}$$

Substitute Eqs. (15) and (16) in Eq. (14) to obtain $(\Delta P_{Li})^t$:

$$(\Delta P_{Li})^t = \left[(\Delta P_L)^t + (\Delta P_l)^t\right](S_i/S_T). \tag{18}$$

Then

$$(\Delta P_{Li})^t = (CP)_i\left[(\Delta P_L)^t + (\Delta P_l)^t\right] \tag{19}$$

where $(CP)_i$ can be defined as the coefficient of participation of unit (i)

$$(CP)_i = S_i/S_T. \tag{20}$$

From Eqs. (6)–(8), we can obtain the following corresponding equations for unit (i) in each time interval (t):

$$(\Delta P_{Li})'^t = D_i(\Delta f)^t \tag{21}$$

$$(\Delta P_{ei})^t = (\Delta P_{Li})^t + (\Delta P_{Li})'^t \tag{22}$$

$$(\Delta P_{mi})^t = -(1/R_i)(\Delta f)^t. \tag{23}$$

Therefore

$$(\Delta P_{mT})^t = \sum_{i=1}^{N}(\Delta P_{mi})^t = -(\Delta f)^t \sum_{i=1}^{N}(1/R_i) \tag{24}$$

$$(\Delta P_{eT})^t = \sum_{i=1}^{N}(\Delta P_{ei})^t = \sum_{i=1}^{N}(\Delta P_{Li})^t + \sum_{i=1}^{N}(\Delta P_{Li})'^t \tag{25}$$

where $(\Delta P_{ei})^t$, $(\Delta P_{mi})^t$, $(\Delta P_{eT})^t$ and $(\Delta P_{mT})^t$ are the changes of electrical and mechanical powers of unit (i) and of the system, totally, in interval (t); $(\Delta P_{Li})^t$ and $(\Delta P_{Li})'^t$ are the changes of the non-frequency sensitive and of the frequency sensitive loads shared by generating unit (i).

The steady state value of the frequency $(f)^t$ and the controlled generation power $(P_i)^t$ of unit (i) can be obtained by

$$(f)^t = (f)^{t-1} + (\Delta f)^t f \tag{26}$$

$$(P_i)^t = (P_i)^{t-1} + (\Delta P_{Li})^t \tag{27}$$

where (f) and $(\Delta f)^t$ are the standard system frequency and the per unit frequency change in interval (t); $(f)^{t-1}$ and $(P_i)^{t-1}$ are the steady state values of the frequency and the controlled generation power in the previous interval $(t-1)$, respectively.

The control of generation holds the system frequency at, or very close to, a specified nominal value and maintains the generation of each generating unit at the most economic value [6–8]. Starting from an optimal economic solution in the first time interval, this suggested mathematical model, which takes the transmission losses into account, can be used to calculate the system frequency, the deviations in each of electrical and mechanical powers in each time interval (t). The optimal generation of each generating unit can be obtained, also by considering the effect of the change in frequency on the net load drawn by the power system. If the transmission losses are not taken into consideration, the mathematical model, which was given in Ref. [6], can be derived.

3. Mathematical model based on an optional specified constant value of the load change (model 2)

When the load change is taken, optionally, as a specified constant value $(\Delta P_{L\ sp})$, the corresponding deviations in frequency, electrical power and mechanical power in addition to the deviation in the power of each generating unit can be obtained from the following proposed mathematical model. Also, the deviations which belong to each required load in each time interval can be calculated. The specified change in the transmission losses $(\Delta P_{l\ sp})$ related to the specified value of the load change can be obtained from the following assumption

$$(\Delta P_{l\,sp}) = \left[(P_{l\max}) - (P_{l\min})\right] / \left\{\left[(P_L)^e - (P_L)^s\right] / (\Delta P_{L\,sp})\right\} \tag{28}$$

where the load ranges from $(P_L)^s$, which must be greater than or equal to the sum of the minimum powers of the generating units, to $(P_L)^e$, which must be less than or equal to the sum of the maximum powers of the units. The equality sign must be taken into account when the

transmission loss are not taken into consideration. ($P_{l\ \min}$) and ($P_{l\max}$) are the transmission losses corresponding to $(P_L)^s$ and $(P_L)^e$, respectively.

The change in the rotational speed ($\Delta\omega_{sp}$) corresponding to the specified changes of load and transmission losses can be given by the following equation, which corresponds to Eq. (17)

$$(\Delta\omega_{sp}) = -\left[(\Delta P_{L\,sp}) + (\Delta P_{l\,sp})\right]/S_T. \tag{29}$$

Therefore, the change in the rotational speed $(\Delta\omega)^t$ at the load $(P_L)^t$ in the time interval (t) can be obtained from

$$(\Delta\omega)^t = \left[(n)^t - (n)^{t-1}\right](\Delta\omega_{sp}) \tag{30}$$

where

$$(n)^t = \left\{\left[(P_L)^t - (P_L)^s\right]/(\Delta P_{L\,sp})\right\} + 1 \tag{31}$$

and $(n)^t$, $(n)^{t-1}$ are the grades of the loads $(P_L)^t$ and $(P_L)^{t-1}$ related to the specified load change.

The frequency $(f)^t$ at the load $(P_L)^t$ in interval (t) can be obtained in terms of the base frequency (f_{base}) by

$$(f)^t = (f_{base}) + \left[(n)^t - 1\right](\Delta\omega_{sp})(f_{base}). \tag{32}$$

The change in mechanical power ($\Delta P_{m\ sp\ i}$) of unit (i) related to the specified load change can be given by

$$(\Delta P_{m\,sp\,i}) = -(\Delta\omega_{sp})/R_i. \tag{33}$$

By substituting Eq. (29) in Eq. (33), we can obtain

$$(\Delta P_{m\,sp\,i}) = \left[(\Delta P_{L\,sp}) + (\Delta P_{l\,sp})\right]/(R_i S_T). \tag{34}$$

In interval (t), the change in mechanical power $(\Delta P_{mi})^t$ of unit (i) related to the load $(P_L)^t$ can be expressed in terms of the specified load change as

$$(\Delta P_{mi})^t = \left[(n)^t - (n)^{t-1}\right](\Delta P_{m\,sp\,i}). \tag{35}$$

Substitute Eq. (34) in Eq. (35) to obtain

$$(\Delta P_{mi})^t = \left[(\Delta P_{L\,sp}) + (\Delta P_{l\,sp})\right]\left[(n)^t - (n)^{t-1}\right]/(R_i S_T). \tag{36}$$

The change in the load $(\Delta P_{L\ sp\ i})'$ due to the change in the system frequency can be given by

$$(\Delta P_{L\,sp\,i})' = (\Delta\omega_{sp})D_i. \tag{37}$$

From Eqs. (29) and (37), we can obtain

$$(\Delta P_{L\,\mathrm{sp}\,i})' = -\Big[(\Delta P_{L\,\mathrm{sp}}) + (\Delta P_{l\,\mathrm{sp}})\Big](D_i/S_T). \tag{38}$$

The change in the controlled generation ($\Delta P_{L\ \mathrm{sp}\ i}$) of unit ($i$) can be obtained, also, by

$$(\Delta P_{L\,\mathrm{sp}\,i}) = -(\Delta\omega_{\mathrm{sp}})\big[(1/R_i) + D_i\big]. \tag{39}$$

From Eqs. (15), (29) and (39), we can obtain

$$(\Delta P_{L\,\mathrm{sp}\,i}) = \Big[(\Delta P_{L\,\mathrm{sp}}) + (\Delta P_{l\,\mathrm{sp}})\Big](S_i/S_T). \tag{40}$$

In time interval (t), the change in electrical power $(\Delta P_{ei})^t$ of unit (i) related to the load $(P_L)^t$ can be expressed, after substituting Eqs. (38) and (40), by

$$\begin{aligned}(\Delta P_{ei})^t &= \big[(n)^t - (n)^{t-1}\big]\big[(\Delta P_{L\,\mathrm{sp}\,i}) + (\Delta P_{L\,\mathrm{sp}\,i})'\big] \\ &= \Big[(\Delta P_{L\,\mathrm{sp}}) + (\Delta P_{l\,\mathrm{sp}})\Big]\big[(n)^t - (n)^{t-1}\big]/(R_i S_T).\end{aligned} \tag{41}$$

Eqs. (36) and (41) satisfy the requirement that the change in mechanical power equals the corresponding change in electrical power. The generated power $(P_i)^t$ of unit (i) in interval (t) can be obtained from the corresponding starting power value $(P_i)^s$ and the specific change in the controlled generation ($\Delta P_{L\ \mathrm{sp}\ i}$) as

$$(P_i)^t = (P_i)^s + \big[(n)^t - 1\big](\Delta P_{L\,\mathrm{sp}\,i}). \tag{42}$$

Substitute from Eq. (40) in Eq. (42) to obtain

$$(P_i)^t = (P_i)^s + \big[(n)^t - 1\big]\Big[(\Delta P_{L\,\mathrm{sp}}) + (\Delta P_{l\,\mathrm{sp}})\Big](S_i/S_T). \tag{43}$$

The transmission losses corresponding to each load in each time interval (t) can be calculated by

$$(P_l)^t = (P_{l\,\mathrm{min}}) + \big[(n)^t - 1\big](\Delta P_{l\,\mathrm{sp}}). \tag{44}$$

This mathematical model consists of Eqs. (28)–(32), (34), (36) and (38) and from Eqs. (40)–(44). To obtain the mathematical model without transmission loss consideration, ($\Delta P_{l\ \mathrm{sp}}$) must be replaced by zero in the previous equations.

4. Test example and results

Table 1 shows the given data of each controlled generating unit of a power system.

The optimization time period equals 24 h (divided into 6 time intervals). Table 2 gives the load $(P_L)^t$ and load change $(\Delta P_L)^t$ for each time interval.

Let the transmission losses (P_l) be expressed by the following equation in per unit on a 100 MVA base (all generation powers must be per unit on 100 MVA base).

Table 1. Cost functions and power limits data

Unit	a_i	b_i	c_i	$P_{i\,max}$	$P_{i\,min}$	D_i	R_i
1	0.00533	11.669	213.1	200.0	50.0	0.8	0.01
2	0.00889	10.333	200.0	150.0	37.5	0.9	0.03
3	0.00741	10.833	240.0	180.0	45.0	1.0	0.02

Table 2. Daily load curve data

Interval (t)	1	2	3	4	5	6
Load (MW)	210.0	375.0	470.0	400.0	320.0	180.0
Load change	0.0	165.0	95.0	−70.0	−80.0	−140.0

Table 3. Generation of the first generating unit $(P_1)^t$

	$(P_1)^t$ without TL		$(P_1)^t$ with TL			
Int. (t)	Model 1a	Model 2a	Model 1b	Model 2b	Model 2c	Model 2 d
1	50.00	87.93	60.27	88.42	92.48	92.31
2	139.40	177.33	158.71	188.48	192.63	192.03
3	190.88	200.00	200.00	200.00	200.00	200.00
4	152.95	190.88	154.56	200.00	200.00	200.00
5	109.60	147.53	105.26	153.95	159.25	158.79
6	50.00	71.67	50.00	71.27	74.28	74.18

Table 4. Generation of the second generating unit $(P_2)^t$

	$(P_2)^t$ without TL		$(P_2)^t$ with TL			
Int. (t)	Model 1a	Model 2a	Model 1b	Model 2b	Model 2c	Model 2 d
1	88.07	57.88	79.45	66.26	67.27	67.21
2	118.43	88.24	112.88	100.24	101.28	101.08
3	135.91	134.53	150.00	150.00	150.00	150.00
4	123.03	92.84	134.57	110.33	114.23	113.35
5	108.31	78.12	117.83	88.52	89.94	89.79
6	82.55	52.36	90.41	60.44	61.08	61.05

$$P_l = [P_1\ P_2\ P_3]\begin{bmatrix} 0.06760 & 0.00953 & -0.00507 \\ 0.00953 & 0.05210 & 0.00901 \\ -0.00507 & 0.00901 & 0.02940 \end{bmatrix}\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$+[-0.07660\ \ -0.00342\ \ 0.01890]\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} + 0.040357.$$

To apply the first mathematical model, in which the load change $(\Delta P_L)^t$ was taken as the difference between the interval load and the previous interval load, take the following generating powers, which were obtained from an economic dispatch with penalty factors [7], for starting in the first interval ($t = 1$), at which the reference load is 210.0 MW:

$$-P_1 = 60.2677 \text{ MW}, \quad P_2 = 79.4462 \text{ MW} \quad \text{and} \quad P_3 = 80.1503 \text{ MW}.$$

The first suggested mathematical model has been applied for two cases (Model 1a and Model 1b), with and without taking the transmission losses (TL) into account.

In the second model, the specified constant load change $(\Delta P_{L\ \mathrm{sp}})$ has been taken as different values (1, 5, 10 and 20 MW). The grade of each required load can be obtained from Eq. (31), depending upon the specified value of the load change and the starting load $(P_L)^s$, which is taken as a reference load. The values of $(P_L)^s$ and $(P_L)^e$ have been taken as 140 and 470 MW, respectively. The generating powers corresponding to $(P_L)^s$ can be taken, for starting, as $-P_1 = 50.0$ MW,

$P_2 = 52.84$ MW and $P_3 = 42.48$ MW.

Starting from $(P_L)^s$ and by increasing this load gradually by the specified value of the load change, the corresponding control variables will be obtained and can be tabulated. The control variables related to the actual required loads, which are given in Table 2, can be, also, calculated directly from Eqs. (30)–(32), (36), (41), (43) and (44). The different values of the specified load change do not affect the values of the obtained control variables and give, approximately, the same results related to the required loads. The second suggested model has been applied for four cases. The transmission losses are neglected in the first case (Model 2a), while in the second case (Model 2b), the losses have been taken into account. The values of $(P_{l\ \min})$ and $(P_{l\ \max})$ have been taken, to obtain the value of $(\Delta P_{L\ \mathrm{sp}})$ (Model 2c), from another solution, as 5.32 and 44.96 MW, respectively. In the fourth case (Model 2 d), it has been suggested that the values of $(P_{l\ \min})$ and $(P_{l\ \max})$ can be taken as $r_1\ (P_L)^s$ and $r_2\ (P_L)^e$, respectively. The constants of proportionality r_1 and r_2 have been taken, optionally, as $r_1 = 3\%$ and $r_2 = 9\%$. The obtained output data are tabulated, for the six cases in each time interval (t), as illustrated herein. Tables 3–5 give the generation of each generating unit. The total generated power $(P_T)^t$, total energy per day in (MW) and the percent error ($\%E$) are given in Table 6. The transmission losses $(P_l)^t$ are given in Table 7.

The corresponding generation cost per hour $(K_T)^t$, total cost per day (TK_T), average cost per hour (AK) and the percent error ($\%E$) are given in Table 8. The percent errors are referred to the results of the first model.

Table 5. Generation of the third generating unit $(P_3)^t$

	$(P_3)^t$ without TL		$(P_3)^t$ with TL			
Int. (t)	Model 1a	Model 2a	Model 1b	Model 2b	Model 2c	Model 2 d
1	71.93	64.19	80.15	63.62	63.98	63.89
2	117.16	109.42	129.96	114.25	114.64	114.34
3	143.21	135.47	164.97	164.94	164.96	163.42
4	124.02	116.28	141.98	122.25	122.32	121.99
5	102.09	94.35	117.04	96.78	97.75	97.52
6	47.45	55.97	48.58	54.94	54.76	54.71

Table 6. Total generated power $(P_T)^t$, MWh/day and the $\%E$

	$(P_T)^t$ without TL		$(P_T)^t$ with TL			
Int. (t)	Model 1a	Model 2a	Model 1b	Model 2b	Model 2c	Model 2 d
1	210.00	210.00	219.86	218.30	223.73	223.41
2	374.99	374.99	401.55	402.97	408.55	407.45
3	470.00	470.00	514.97	514.94	514.96	513.42
4	400.00	400.00	431.11	432.57	436.55	435.34
5	320.00	320.00	340.13	339.25	346.94	346.10
6	180.00	180.00	188.99	186.66	190.12	189.94
MWh	7820.0	7820.0	8386.4	8378.8	8483.4	8462.6
$\%E$	—	0.00	—	−0.09	1.16	0.91

Table 7. The transmission losses $(P_l)^t$

	$(P_l)^t$ without TL		$(P_l)^t$ with TL			
Int. (t)	Model 1a	Model 2a	Model 1b	Model 2b	Model 2c	Model 2 d
1	—	—	9.87	8.31	13.73	12.28
2	—	—	26.55	27.98	33.55	31.33
3	—	—	44.98	44.96	44.96	42.30
4	—	—	31.11	32.59	36.55	34.22
5	—	—	20.13	19.26	26.94	24.98
6	—	—	8.99	6.66	10.12	8.82

Table 8. Total generation costs $(K_T)^t$, (TK_T), (AK) and $(\%E)$

	$(K_T)^t$ without TL		$(K_T)^t$ with TL			
Int. (t)	Model 1a	Model 2a	Model 1b	Model 2b	Model 2c	Model 2 d
1	3046.41	3074.12	3168.62	3169.48	3236.58	3232.59
2	5102.79	5145.17	5451.95	5501.31	5575.66	5561.28
3	6346.61	6354.63	6938.92	6938.41	6938.74	6918.30
4	5425.88	5470.47	5822.94	5883.36	5932.40	5917.36
5	4402.68	4440.16	4650.82	4677.98	4777.58	4766.82
6	2694.16	2711.74	2800.51	2786.42	2829.08	2826.83
TK_T	108,074.1	108,785.2	115,335.1	115,827.9	117,160.2	116,892.7
AK	4503.09	4532.72	4805.63	4826.16	4881.67	4870.53
$\%E$	—	0.658	—	0.427	1.582	1.350

Table 9. Change in mechanical (electrical) power of the first generating unit $(\Delta P_{m1})^t$

	$(\Delta P_{m1})^t$ without TL		$(\Delta P_{m1})^t$ with TL			
Int. (t)	Model 1a	Model 2a	Model 1b	Model 2b	Model 2c	Model 2 d
1	112.88	37.63	118.19	39.20	42.15	41.97
2	88.69	88.69	97.66	99.26	99.35	98.93
3	51.07	51.07	61.24	61.62	57.20	56.96
4	−37.63	−37.63	−45.08	−45.62	−42.15	−41.97
5	−43.01	−43.01	−48.90	−50.25	−48.17	−47.97
6	−75.26	−75.26	−80.09	−82.02	−84.30	−83.94

Table 10. Change in mechanical (electrical) power of the second generating unit $(\Delta P_{m2})^t$

	$(\Delta P_{m2})^t$ without TL		$(\Delta P_{m2})^t$ with TL			
Int. (t)	Model 1a	Model 2a	Model 1b	Model 2b	Model 2c	Model 2 d
1	37.63	12.54	39.40	13.07	14.05	13.99
2	29.57	29.57	32.55	33.09	33.12	32.98
3	17.02	17.02	20.41	20.54	19.07	18.99
4	−12.54	−12.54	−15.03	−15.21	−14.05	−13.99
5	−14.33	−14.33	−16.30	−16.75	−16.06	−15.99
6	−25.09	−25.09	−26.70	−27.34	−28.10	−27.98

Table 11. Change in mechanical (electrical) power of the third generating unit $(\Delta P_{m3})^t$

	$(\Delta P_{m3})^t$ without TL		$(\Delta P_{m3})^t$ with TL			
Int. (t)	Model 1a	Model 2a	Model 1b	Model 2b	Model 2c	Model 2 d
1	56.44	18.81	59.09	19.60	21.07	20.99
2	44.35	44.35	48.83	49.63	49.67	49.47
3	25.53	25.53	30.62	30.81	28.60	28.48
4	−18.81	−18.81	−22.54	−22.81	−21.07	−20.99
5	−21.50	−21.50	−24.45	−25.13	−24.08	−23.98
6	−37.63	−37.63	−40.05	−41.01	−42.15	−41.97

Table 12. Frequency $(f)^t$

	$(f)^t$ without TL		$(f)^t$ with TL			
Int. (t)	Model 1a	Model 2a	Model 1b	Model 2b	Model 2c	Model 2 d
1	50.000	49.962	50.000	49.961	49.958	49.958
2	49.911	49.874	49.902	49.862	49.859	49.859
3	49.860	49.823	49.841	49.800	49.801	49.802
4	49.898	49.860	49.886	49.846	49.844	49.844
5	49.941	49.903	49.935	49.896	49.892	49.892
6	50.016	49.978	50.015	49.978	49.976	49.976

Table 13. The change per unit frequency $(\Delta\omega)^t$

	$(\Delta\omega)^t$ without TL		$(\Delta\omega)^t$ with TL			
Int. (t)	Model 1a	Model 2a	Model 1b	Model 2b	Model 2c	Model 2 d
1	0.00000	−0.00075	0.00000	−0.00079	−0.00084	−0.00084
2	−0.00178	−0.00178	−0.00195	−0.00199	−0.00199	−0.00198
3	−0.00102	−0.00102	−0.00123	−0.00123	−0.00114	−0.00114
4	0.00075	0.00075	0.00090	0.00091	0.00084	0.00084
5	0.00086	0.00086	0.00098	0.00101	0.00096	0.00096
6	0.00150	0.00150	0.00160	0.00164	0.00169	0.00168

The changes in mechanical power (which equals the changes in electrical power) of each generating unit are given in Tables 9–11, respectively. The frequency $(f)'$ and the change per unit frequency $(\Delta\omega)'$ in each time interval are given in Table 12 and 13.

5. Conclusions

The feasibility of the two proposed procedures is tested through six different cases of application on a given thermal power system. The approximate agreement between the results obtained (generation powers and costs, transmission losses, system frequency and change in mechanical and electrical powers) verifies the two presented models, which need small computing time and small storage. Although a small difference in values of the power generated by each unit in some intervals has been found, the total MWh and the total generation costs per day are, approximately, the same. It is clear that the two cases, in which the transmission losses are not taken into account and also, the fourth case of the second model (model 2 d), which is based on some approximations, are more suitable for on-line calculations. The applications of the second proposed model has an important advantage that tabulated results for a very wide range of loads can be obtained, specially for the smaller values of the optional specified load change.

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